GINF5020, Spring 2007
Homework 1
Due Week of January 15 in class

1. Prove: $\forall n \geq 1, \sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$.
2. Prove: $\forall n \geq 0, \sum_{i=0}^{n} 3^i = (3^{n+1}-1)/2$.
3. This is a variation of the Tower of Hanoi problem. The tower has $2n$ disks: there are two disks for each size, and there are n different sizes.
   (a) $H(1) = 2$. Find the recurrence relation between $H(n)$ and $H(n-1)$.
   (b) The closed form of the recurrence relation is: $H(n) = 2^{n+1} - 2$. Prove by mathematical induction that the closed form is correct.
4. There are n straight lines in general position on the plane. Let $f(n)$ be the number of (finite or infinite) line segments.
   (a) $f(1) = 1$, $f(2) = 4$, and $f(3) = 9$. Find the recurrence relation between $f(n)$ and $f(n-1)$.
   (b) The closed form of the recurrence relation is: $f(n) = n^2$. Prove by mathematical induction that the closed form is correct.